

USN

--	--	--	--	--	--	--	--	--	--

10MAT31

**Third Semester B.E. Degree Examination, June/July 2017**  
**Engineering Mathematics - III**

Time: 3 hrs.

Max. Marks:100

**Note:** Answer any FIVE full questions, selecting atleast TWO questions from each part.

PART - A

1 a. Obtain Fourier series for the function  $f(x)$  given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi \end{cases}$$

Hence deduce that  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$ . (06 Marks)

b. Obtain Fourier half range Cosine series for the function  $f(x) = x \sin x$  in  $(0, \pi)$ . Hence show that  $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots \infty = \frac{\pi - 2}{4}$ . (07 Marks)

Obtain the constant term and the co-efficient of the first sine and cosine terms in the Fourier series of  $f(x)$  as given in the following table. (07 Marks)

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

2 a. Find the Fourier transform of  $e^{-a^2x^2}$ ,  $a < 0$ . Hence deduce that  $e^{-x^2/2}$  is self reciprocal in respect of Fourier transform. (06 Marks)

b. Find the Fourier sine transform of  $e^{-|x|}$ . Hence show that  $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}$ ,  $m > 0$ . (07 Marks)

c. Find the Fourier Cosine transform of  $f(x) = \frac{1}{1+x^2}$ . (07 Marks)

3 a. Obtain various possible solutions of the one dimensional Heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

by the method of separation of variables. (06 Marks)

b. Obtain the D'Alembert's solution of the wave equation  $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ . Subject to the conditions  $u(x, 0) = f(x)$  and  $\frac{\partial u}{\partial t}(x, 0) = 0$ . (07 Marks)

c. Obtain various possible solutions of the two dimensional Laplace equation  $u_{xx} + u_{yy} = 0$  by the method of separation of variables. (07 Marks)

4 a. Fit a parabola  $y = ax^2 + bx + c$  to the following data : (06 Marks)

x	0	1	2	3	4	5
y	1	3	7	13	21	31

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8=50, will be treated as malpractice.

- b. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 5,760 to invest and has space for at most 20 items. A fan and sewing machine cost Rs 360 and Rs 240 respectively. He can sell a fan at a profit of Rs 22 and sewing machine at a profit of Rs 18. Assuming that he can sell whatever he buys, how should he invest his money in order to maximize his profit? Translate the problem into LPP and solve it graphically. (07 Marks)

- c. Use Simplex method to solve the following LPP

$$\text{Minimize } Z = x_1 - 3x_2 + 3x_3$$

$$\text{Subject to } 3x_1 - x_2 + 2x_3 \leq 7$$

$$2x_1 + 4x_2 \geq -12$$

$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0.$$

(07 Marks)

**PART - B**

- 5 a. Using Newton - Raphson method, find the value of  $\sqrt[3]{18}$  correct to 2 decimals, assuming 2.5 as the initial approximation. (06 Marks)

- b. Apply Gauss - Seidal iteration method to solve the following equations :

$$3x + 20y - z = -18 ; \quad 2x - 3y + 20z = 25 ; \quad 20x + y - 2z = 17. \quad (07 \text{ Marks})$$

- c. Find the largest Eigen - value and the corresponding Eigen - vector for the matrix

$$\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$$

with initial approximation  $[1 \ 1 \ 0]^T$ .

(07 Marks)

- 6 a. Determine  $f(x)$  as a polynomial in  $x$  for the following data by using Newton's divided difference formula. (06 Marks)

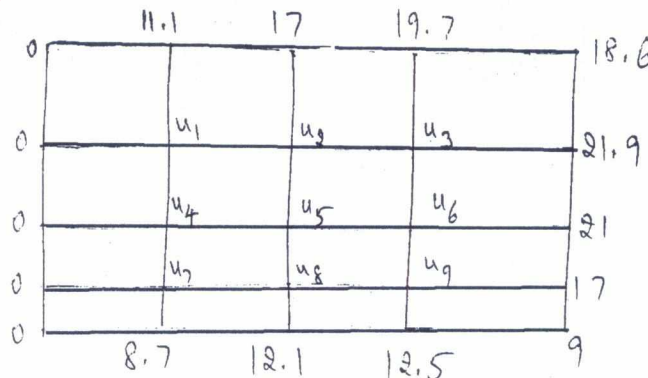
x	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

- b. From the data given in the following table, find the number of students who obtained  
i) less than 45 marks and ii) between 40 and 45 marks. (07 Marks)

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

- c. Evaluate  $\int_4^{5.2} \log_e x \, dx$  by Weddle's rule. (07 Marks)

- 7 a. Solve the Laplace equation  $u_{xx} + u_{yy} = 0$ , given that the boundary values for the following square mesh. (06 Marks)



- b. Evaluate the pivotal values of the equation  $u_{tt} = 16u_{xx}$ , taking  $h = 1$  upto  $t = 1.25$ . The boundary conditions are  $u(0,t) = u(5,t) = 0$ ,  $u_i(x, 0) = 0$  and  $u(x, 0) = x^2(5-x)$ . (07 Marks)
- c. Solve  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$  in  $0 < x < 5$ ,  $t \geq 0$ , given that  $u(x, 0) = 20$ ,  $u(0, t) = 0$ ,  $u(5, t) = 100$ . Compute  $u$  for the time - step with  $h = 1$  by Crank - Nicholson method. (07 Marks)
- 8 a. Find the Z - transform of the following :
- i)  $(n + 1)^2$  ii)  $\sin(3n + 5)$  iii)  $n_c^p$  ( $0 \leq p \leq n$ ). (06 Marks)
- b. If  $u(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$ . Find  $u_0, u_1, u_2, u_3$ . (07 Marks)
- c. Solve  $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$  with  $y_0 = 0$ ,  $y_1 = 1$ , using Z - transforms. (07 Marks)

\*\*\*\*\*

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

**Third Semester B.E. Degree Examination, June/July 2017**  
**Analog Electronic Circuits**

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting at least TWO questions from each part.**

**PART – A**

- 1 a. With respect to a semiconductor diode, explain the following:
  - (i) Reverse recovery time
  - (ii) Diffusion capacitance. (06 Marks)
- b. Explain the working of full wave bridge rectifier and derive the expression for ripple factor and efficiency. (08 Marks)
- c. Design an ideal clamper circuit to obtain the output waveform as shown in Fig. Q1 (c) for the given input. (06 Marks)

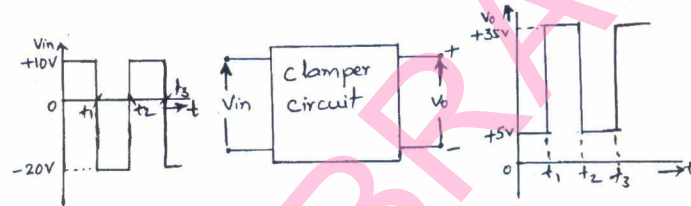


Fig. Q1 (c)

- 2 a. Explain with help of load line the effect of variation of  $V_{CC}$ ,  $I_B$  on Q-point of a transistor. (06 Marks)
- b. Derive the expression for stability factors for voltage divider bias circuit with respect to  $I_{CO}$ ,  $V_{BE}$  and  $\beta$ . (06 Marks)
- c. Determine the voltage  $V_{CE}$  and the current  $I_C$  for the voltage divider configuration shown in Fig. Q2 (c) (08 Marks)

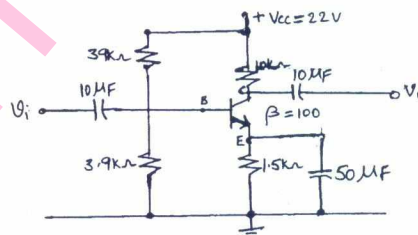


Fig. Q2 (c)

- 3 a. Draw the re-equivalent circuit of CE fixed bias configuration and derive the expression for  $Z_{in}$ ,  $Z_O$  and  $A_v$ . (10 Marks)
- b. What are the advantages of h-parameters? (04 Marks)
- c. For the network shown in Fig. Q3 (c), determine  $r_e$ ,  $Z_i$ ,  $Z_o$ ,  $A_v$ . (06 Marks)

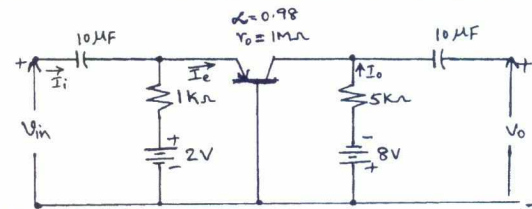


Fig. Q3 (c)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- 4 a. Obtain expression for Miller effect input and Miller effect output capacitance. (06 Marks)  
 b. Draw and discuss the effect of various capacitors on high frequency response. (06 Marks)  
 c. Determine the lower cutoff frequency for the voltage divider bias BJT amplifier with  $C_S = 10 \mu\text{F}$ ,  $C_C = 1 \mu\text{F}$ ,  $C_E = 20 \mu\text{F}$ ,  $R_S = 1 \text{ K}\Omega$ ,  $R_1 = 40 \text{ K}\Omega$ ,  $R_2 = 10 \text{ K}\Omega$ ,  $R_E = 2 \text{ K}\Omega$ ,  $R_C = 4 \text{ K}\Omega$ ,  $R_L = 2.2 \text{ K}\Omega$ ,  $\beta = 100$ ,  $r_o = \infty \Omega$ ,  $V_{CC} = 20\text{V}$  (08 Marks)

**PART – B**

- 5 a. Explain the important advantages of a negative feedback amplifier. (04 Marks)  
 b. Obtain expression for  $Z_{if}$  and  $Z_{of}$  for voltage series feedback amplifier. (08 Marks)  
 c. Why do we cascade amplifier? State the various method of cascading transistor amplifier. A given amplifier arrangements has the following voltage gains.  $A_{V_1} = 10$ ,  $A_{V_2} = 20$  and  $A_{V_3} = 40$ . What is the overall voltage gain? Also express each gain in dB and determine the total voltage gain in dB? (08 Marks)
- 6 a. With a neat circuit diagram, explain the operation of a transformer coupled class A power amplifier. (06 Marks)  
 b. Prove that the maximum conversion efficiency in class B power amplifier is 78.5%. (08 Marks)  
 c. A power amplifier has harmonic distortions  $D_2 = 0.1$ ,  $D_3 = 0.02$ ,  $D_4 = 0.01$ , the fundamental current  $I_1 = 4$  Amps and  $R_L = 8 \Omega$ . Calculate the total harmonic distortion, fundamental power and total power. (06 Marks)
- 7 a. State Barkhausen criteria for sustained oscillations apply this to a transistorized Weinbridge oscillator and explain its operation. (10 Marks)  
 b. Explain the working of BJT Colpitt's oscillator. (06 Marks)  
 c. Calculate the frequency of oscillations of a Colpitt's oscillator,  $L = 100 \mu\text{H}$ ,  $C_1 = 100 \text{ pF}$ ,  $C_2 = 1000 \text{ pF}$ . (04 Marks)
- 8 a. Derive expression for  $V_{GSQ}$ ,  $I_{DQ}$ ,  $V_{DS}$ ,  $V_S$ ,  $V_G$  and  $V_D$  for a self bias JFET circuit. (10 Marks)  
 b. Determine  $I_{DQ}$ ,  $V_{GSQ}$  and  $V_{DS}$  for the P-channel JFET of Fig. Q8 (b). (10 Marks)

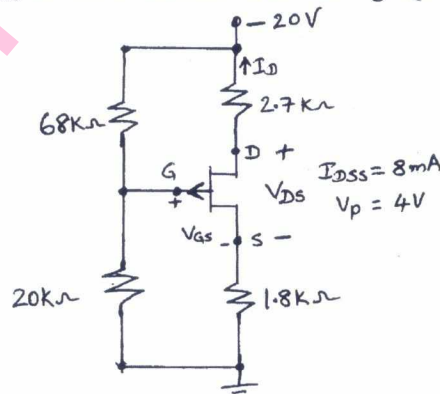


Fig. Q8 (b)

\*\*\*\*\*

**Third Semester B.E. Degree Examination, June/July 2017**  
**Network Analysis**

Time: 3 hrs.

Max. Marks: 100

**Note: Answer FIVE full questions, selecting at least TWO questions from each part.**

**PART – A**

- 1 a. Calculate the current through  $2\Omega$  resistor in the network shown in Fig. Q1 (a) by source transformation method. (06 Marks)
- b. Compute the resistance across the terminals A and B of the network shown in Fig. Q1(b) by star delta transformation. (06 Marks)
- c. Use mesh analysis to determine what value of  $V_2$  in the network shown in Fig. Q1(c). Cause voltage  $V = 0$  across  $20\Omega$  resistor. (08 Marks)

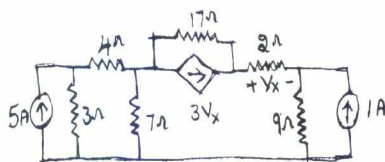


Fig. Q1(a)

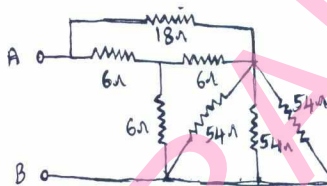


Fig. Q1(b)

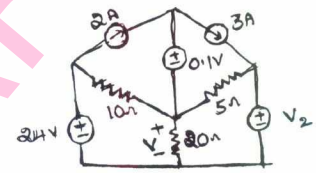


Fig. Q1(c)

- 2 a. Define with examples :  
 i) oriented graph    ii) Tree    iii) Cut set matrix    iv) Tie set matrix. (08 Marks)
- b. For the network shown in Fig. Q2(b) draw the graph. Select 2 and 4 as tree branches. Draw the tie set matrix. Write down the equilibrium equations with loop currents as variables. Solve these equations and find the various branch voltages and currents. The integers indicate branch numbers. Use matrix method. (08 Marks)
- c. Draw the dual of the network shown in Fig. Q2(c). (04 Marks)

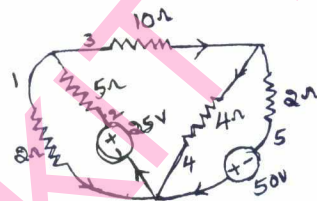


Fig. Q2(b)

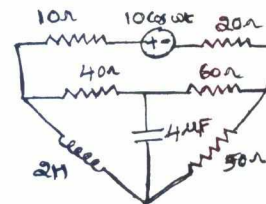


Fig. Q2(c)

- 3 a. Find  $V_a$  using superposition principle in the circuit shown in Fig. Q3(a). (08 Marks)
- b. In the single current source circuit shown in Fig. Q3(b), find the voltage  $V_x$ . Interchange the current source and the resulting voltage  $V_x$ . Is the Reciprocity theorem verified? (06 Marks)

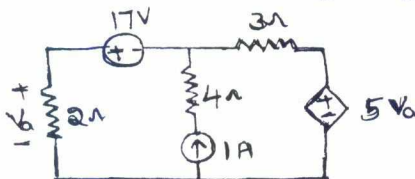


Fig. Q3(a)

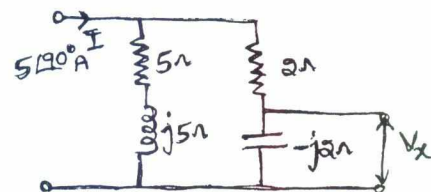


Fig. Q3(b)

- c. State and explain Millman's theorem. (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- 4 a. For the network shown in Fig. Q4(a) , obtain the Thevinin's equivalent as seen from terminals p and q. (08 Marks)  
 b. Obtain Norton's equivalent circuit for the network shown in Fig. Q4(b). (06 Marks)

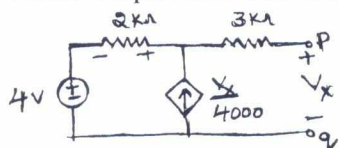


Fig. Q4(a)

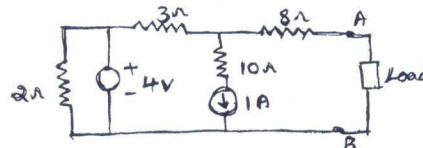


Fig. Q4(b)

- c. Prove that an alternating voltage source transfers maximum power to a load when the load impedance is the conjugate of the source impedance. (06 Marks)

**PART - B**

- 5 a. Define quality factor and bandwidth. Also establish the relationship between them in a series resonance circuit. (08 Marks)  
 b. Show that resonant frequency of series resonance circuit is equal to the geometric mean of two half power frequencies. (06 Marks)  
 c. Find the value of  $R_L$  for which the circuit shown in Fig. Q5(c) is resonant. (06 Marks)



Fig. Q5(c)

- 6 a. Show that  
 i) The voltage of a capacitor cannot change instantaneously  
 ii) The current in an inductor cannot change instantaneously. (10 Marks)  
 b. In the circuit of Fig. Q6(b). Switch K is changed from 1 to 2 at  $t = 0$  steady state having been attained in position 1. Find the values of  $i$ ,  $\frac{di}{dt}$  and  $\frac{d^2i}{dt^2}$  at  $t = 0$ . (10 Marks)

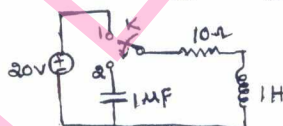


Fig. Q6(b)

- 7 a. State and prove i) Initial value theorem and ii) Final value theorem. (10 Marks)  
 b. Determine the response current  $i(t)$  in the circuit shown in Fig. Q7(b). Using Laplace transform. (10 Marks)

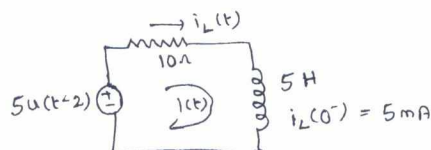


Fig. Q7(b)

- 8 a. Explain Z and Y parameters with equivalent circuit Also express Z parameters in terms of Y parameters. (10 Marks)  
 b. Obtain the Y parameters of the two port network shown in Fig. Q8(b). (10 Marks)

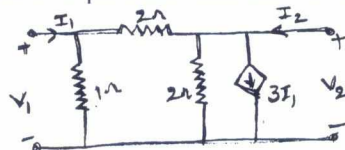


Fig. Q8(b)

\*\*\*\*\*

--	--	--	--	--	--	--	--	--	--

**Third Semester B.E. Degree Examination, June/July 2017**  
**Electronic Instrumentation**

Time: 3 hrs.

Max. Marks: 100

**Note: 1. Answer any FIVE full questions, selecting  
atleast TWO questions from each part.  
2. Missing data, if any, may be suitably assumed.**

**PART – A**

- 1 a. What is systematic error? Explain the different types of systematic errors and how they can be avoided? (08 Marks)
- b. The value of voltage across a resistor is 5V. The voltmeter reads as 4.95V. Calculate :  
i) Absolute error ii) % error iii) relative accuracy iv) % of accuracy. (04 Marks)
- c. Draw the block diagram of a true RMS voltmeter and explain the working. (08 Marks)
- 2 a. Discuss the operating and performance characteristics of digital voltmeter. (04 Marks)
- b. With the help of block diagram and waveforms, explain the principle of operation of integrating type DVM which converts voltage to frequency. (08 Marks)
- c. Draw the block diagram of a basic digital multimeter and explain the working. (08 Marks)
- 3 a. Write the basic CRO block diagram and explain the function of each of the blocks. (08 Marks)
- b. Explain with waveforms the : i) ALTERNATE mode and ii) CHOP mode of operation in a dual trace oscilloscope. (08 Marks)
- c. What is the function of electronic switch? Explain with basic block diagram. (04 Marks)
- 4 a. What is the need of delayed time Basic system? Explain. (04 Marks)
- b. With the help of block diagram and waveforms explain the working of a sampling oscilloscope. (10 Marks)
- c. Discuss the applications of digital storage oscilloscope. (06 Marks)

**PART – B**

- 5 a. Draw the block diagram of AF sine and square wave generator and explain its working in detail. (08 Marks)
- b. Explain the principle of operation of frequency synthesizer using PLL system. (08 Marks)
- c. Describe briefly the sweep frequency generator. Also mention its applications. (04 Marks)
- 6 a. What are the limitations of wheat-stone bridge? (04 Marks)
- b. Given a centre zero (200 – 0 – 200) MA movement having an internal resistance of 125Ω. Calculate the current through the galvanometer by the approximate method for the wheat-stones bridge with four arms as 700Ω, 700Ω 700Ω and 735Ω, and E = 10V. (06 Marks)
- c. A Maxwell bridge is used to measure inductance. The bridge constants at balance are :  
Find the series equivalent of the unknown impedance. Derive the relations used. (10 Marks)
- 7 a. What are the main advantages of electrical transducers? Explain in brief. (04 Marks)
- b. Discuss the construction of semiconductor strain gage and list the advantages and disadvantages (08 Marks)
- c. Explain the principle of operation of LVDT with the help of neat sketch. (08 Marks)
- 8 a. With circuit diagram and characteristics, explain the principle of operation of photo-transistor. (08 Marks)
- b. Write a note on classification of displays. (04 Marks)
- c. With a neat diagram, explain the measurement of RF power using bolometer bridge. (08 Marks)

\* \* \* \* \*





### Third Semester B.E. Degree Examination, June/July 2017

### Field Theory

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

#### PART – A

1. a. Drive the expression for electric field intensity [EFI] due to infinite line charge. (08 Marks)  
 b. A point charge  $Q_1 = 25 \text{ nc}$  is located at  $P_1(4, -2, 7)$  and a charge  $Q_2 = 60 \text{ nc}$  is at  $P_2(-3, 4, 2)$ , if  $\epsilon = \epsilon_0$ , find: i)  $\vec{E}$  at  $P_3(1, 2, 3)$  and (ii) At which point on the 'y' axis is  $E_x = 0$ ? (06 Marks)  
 c. A cube is defined by  $1 < x, y, t < 1.2$ . If  $\vec{D} = 2x^2y \vec{a}_x + 3x^2y^2 \vec{a}_y \text{ c/m}^2$ .  
 i) Apply Gauss's law to find the total flux leaving the closed surface of the cube.  
 ii) Evaluate  $\nabla \cdot \vec{D}$  at the centre of the cube.  
 iii) Estimate the total charge enclosed within the cube. (06 Marks)
2. a. Find the workdone in moving a charge of  $+2\text{C}$  from  $(2, 0, 0)\text{m}$  to  $(0, 2, 0)\text{m}$  along the straight line path joining the two points. If the electric field is  $\vec{E} = 2x \vec{a}_x - 4y \vec{a}_y \text{ V/m}$ . (08 Marks)  
 b. Prove that  $\vec{E} = -\nabla V$ . (06 Marks)  
 c. A potential field in free space is expressed as  $V = \frac{20}{xyz}$  volts.  
 i) Find the total energy shred within the cube  $1 < x, y, z < 2$ .  
 ii) What value of the energy would be obtained by assuming a uniform energy density equal to the value at the centre of the cube? (06 Marks)
3. a. Let  $\epsilon = \epsilon_0$ , and  $V = 90 Z^{\frac{4}{3}}$  in the region  $z = 0$ .  
 i) Obtain expression for  $\vec{E}$ ,  $\vec{D}$  and  $\rho_v$  as function of  $\vec{Z}$ .  
 ii) If the velocity of charge density is given as  $V_z = 5 \times 10^{-6} Z^{\frac{2}{3}} \text{ m/s}$ . Find  $I_z$  at  $z = 0$  and  $z = 0.1 \text{ m}$ . (06 Marks)  
 b. Derive boundary condition at a boundary between two dielectric medium. (08 Marks)  
 c. Determine whether or not the following potential fields satisfy the Laplace equation:  
 i)  $V = x^2 - y^2 + z^2$       ii)  $V = r \cos \phi + z$  (06 Marks)
4. a. Find the magnetic field intensity ( $\vec{H}$ ) due to straight conductor of finite length using Biot-Savart law. (06 Marks)  
 b. Using Biot-Savart law, find the value of  $\vec{H}$  at that point p for the current circuit shown in Fig.Q4(b).

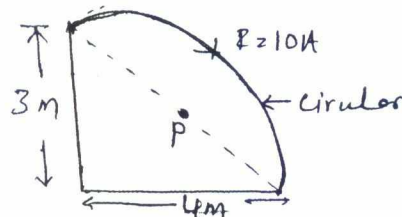


Fig.Q4(b)

- c. Define and derive expression for scalar magnetic potential and vector magnetic potential. (08 Marks)

**PART – B**

- 5 a. Explain force between differential current elements. (06 Marks)  
 b. Explain the magnetic boundary conditions. (08 Marks)  
 c. Define self inductance. Derive expression for the inductance of a co-axial cable. (06 Marks)
- 6 a. Derive Maxwell's equation in vector differential form for time varying field starting from Faraday's law. (06 Marks)  
 b. Derive an expression for general wave equation in free space. (08 Marks)  
 c. An uniform plane of 1 MHz is propagating in medium for which  $\sigma = 5.8 \times 10^7 \text{ } \Omega/\text{m}$  and  $\epsilon_r = \mu_r = 1$ . Find the following:  
 i) Attenuation constant  
 ii) Phase shaft constant  
 iii) Velocity  
 iv) Wavelength (06 Marks)
- 7 a. State and prove Poynting theorem. (10 Marks)  
 b. The region  $Z < 0$  is characterized by  $\epsilon'_R = \mu_R = 1$  and  $\epsilon''_R = 0$ . The total  $\bar{E}$  filed here is given as the sum of the two uniform plane waves:  $E_s = 150e^{-j10z}\bar{a}_x + (50 \angle 20^\circ)e^{j10z}\bar{a}_x \text{ V/m}$ .  
 Find:  
 i) What is the operating frequency?  
 ii) Specify the intrinsic impedance of the region  $Z > 0$  that would provide the appropriate reflected wave.  
 iii) At what value of  $Z(-10\text{cm} < z < 0)$  is the total electric field intensity a maximum amplitude? (10 Marks)
- 8 a. Define SWR and derive the expression for SWR in term of reflection coefficient. (10 Marks)  
 b. Explain reflection of uniform plane waves at normal incidence, derive the expressions for transmission and reflection coefficient. (10 Marks)

\* \* \* \* \*

USN

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

MATDIP301

**Third Semester B.E Degree Examination, June/July 2017**  
**Advanced Mathematics – I**

Time: 3 hrs.

Max. Marks:100

*Note: Answer any FIVE full questions.*

- 1 a. Express :  $\frac{1}{(2+i)^2} - \frac{1}{(2-i)^2}$  in the form of  $a + i b$ . (07 Marks)
- b. Find the modulus and amplitude of the complex number  $1 - \cos \alpha + i \sin \alpha$ . (06 Marks)
- c. Express the complex number  $\sqrt{3} + i$  in the polar form. (07 Marks)
- 2 a. Find the  $n^{\text{th}}$  derivative of  $\log (ax + b)$ . (07 Marks)
- b. Find the  $n^{\text{th}}$  derivative of  $\frac{x}{(x-1)(2x+3)}$ . (06 Marks)
- c. If  $y = \sin^{-1} x$ , prove that :  $(1-x^2)y_{n+2} - (2n+1)x y_{n+1} - n^2 y_n = 0$ . (07 Marks)
- 3 a. Using Taylor's theorem, expand  $\sin x$  in power of  $(x - \pi/2)$ . (07 Marks)
- b. Obtain the Maclaurin's series expansion of the function  $\sqrt{1+\sin 2x}$  up to the term containing  $x^4$ . (06 Marks)
- c. State and prove Euler's theorem. (07 Marks)
- 4 a. Find the total derivative of  $z = xy^2 + x^2y$  where  $x = at$ ,  $y = 2at$ , and also verify the result by direct substitution. (07 Marks)
- b. If  $u = f(y - z, z - x, x - y)$  prove that :  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ . (06 Marks)
- c. if  $x = u(1 - v)$  and  $y = uv$ , find  $J = \frac{\partial(x, y)}{\partial(u, v)}$  and  $J' = \frac{\partial(u, v)}{\partial(x, y)}$  and also verify  $J \cdot J' = 1$ . (07 Marks)
- 5 a. Obtain the reduction formula for  $\int \cos^n x \cdot dx$ . (07 Marks)
- b. Evaluate :  $\int_0^2 \frac{x^4}{\sqrt{4-x^2}} \cdot dx$ . (06 Marks)
- c. Evaluate :  $\int_1^2 \int_1^3 xy^2 dx dy$ . (07 Marks)
- 6 a. Evaluate :  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} x y z dz dy dx$ . (07 Marks)
- b. Prove that  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ . (06 Marks)
- c. Prove that  $\beta(m, n) = \frac{\Gamma_m \Gamma_n}{\Gamma(m+n)}$ . (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- 7 a. Solve :  $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ . (07 Marks)
- b. Solve  $x^2y dx - (x^3 + y^3) dy = 0$ . (06 Marks)
- c. Solve  $\frac{dy}{dx} + y \cot x = \cos x$ . (07 Marks)
- 8 a. Solve :  $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 4y = 0$ . (05 Marks)
- b. Solve  $\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 9y = 3e^{-4x}$ . (05 Marks)
- c. Solve :  $y'' + 2y' + y = e^{-x} + \cos 2x$ . (05 Marks)
- d. Solve :  $\frac{d^2y}{dx^2} - 4y = x \sin 2x$ . (05 Marks)

\* \* \* \* \*